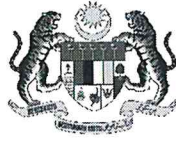**KEMENTERIAN PENDIDIKAN TINGGI**
JABATAN PENDIDIKAN POLITEKNIK DAN KOLEJ KOMUNITI

BAHAGIAN PEPERIKSAAN DAN PENILAIAN
JABATAN PENDIDIKAN POLITEKNIK DAN KOLEJ KOMUNITI
KEMENTERIAN PENDIDIKAN TINGGI

JABATAN MATEMATIK SAINS DAN KOMPUTER

PEPERIKSAAN AKHIR

SESI II : 2022/2023

DBM30033: ENGINEERING MATHEMATICS 3

TARIKH : 08 JUN 2023
MASA : 11.15 PG – 01.15 PTG (2 JAM)

Kertas ini mengandungi **SEMBILAN (9)** halaman bercetak.

Struktur (4 soalan)

Dokumen sokongan yang disertakan : Formula

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIARAHKAN

(CLO yang tertera hanya sebagai rujukan)

INSTRUCTION:

This section consists of **FOUR (4)** structured questions. Answer **ALL** questions.

ARAHAN:

Bahagian ini mengandungi EMPAT (4) soalan berstruktur. Jawab SEMUA soalan.

QUESTION 1

SOALAN 1

CLO1   (a)   Table 1(a) below shows the marks obtained by a group of students in a quiz. Solve the following by using formula and give the answer correct to **TWO** decimal places.

Jadual 1(a) di bawah menunjukkan markah kuiz yang diperolehi oleh sekumpulan pelajar. Selesaikan yang berikut menggunakan formula dan nyatakan jawapan betul sehingga DUA tempat perpuluhan

Table 1(a) / *Jadual 1(a)*

Marks	No of Students
5-9	4
10-14	9
15-19	16
20-24	12
25-29	6
30-34	3

   i.   Calculate mean

         Kira min

                                                                    [5 marks]

                                                                    [*5 markah*]

   ii.   Calculate Median

         Kira median

                                                                    [5 marks]

                                                                    [*5 markah*]

CLO1 | (b)   Given the mean of the data (*x*-2), (*x*+5), (*x*+3), (3*x*-4), (2*x*+7) is 7.

Diberi min bagi data (x+2), (x+5), (x+3), (3x-4), (2x+7) ialah 7

    i.   Determine the value of x

       Tentukan nilai x

[4 marks]

[*4 markah*]

    ii.   Then, identify the median and the mode of the data

       Kemudian, kenalpasti median dan mod bagi data tersebut

[4 marks]

[*4 markah*]

CLO1 | (c)   i.   A box contains of 20 red marbles, 32 blue marbles, 17 yellow marbles and 11 white marbles. A marble is picked randomly from the box. Calculate the probability of picking a red marble.

Sebuah kotak mengandungi 20 guli merah, 32 guli biru, 17 guli kuning dan 11 guli putih. Sebiji guli dipilih secara rawak. Kira kebarangkalian untuk memilih sebiji guli merah.

[2 marks]

[*2 markah*]

    ii.   Ten pieces of paper numbered from 21 to 30 are placed in a file. A piece of paper is picked at random from the file. Calculate the probability of picking a number that is even OR divisible by 3.

Sepuluh keping kertas bernombor 21 hingga 30 diletakkan di dalam sebuah fail. Sekeping kertas dipilih secara rawak dari dalam fail. Kira kebarangkalian untuk memperolehi nombor genap ATAU nombor yang boleh dibahagi dengan 3.

[5 marks]

[*5 markah*]

QUESTION 2

SOALAN 2

CLO1   (a)   Based on the following linear equations:

Berdasarkan persamaan linear di bawah;

$$2x - 2y + 4z = 10$$
$$x + 5y - 4z = 25$$
$$3x - y - 2z = 15$$

i.   Construct Matrix L and Matrix U using Crout Method.

Bina Matriks L dan Matriks U menggunakan Kaedah Crout.

[10 marks]

[*10 markah*]

ii.   Then, calculate the value of x, y and z.

Kemudian, kira nilai x, y dan z.

[8 marks]

[*8 markah*]

CLO1   (b)   Use the Fixed-Point Iteration Method to determine the root for equation $f(x) = x^3 - 5x + 1$ when $x_0 = 2$. Give your answer correct to 2 decimal places.

Gunakan Kaedah Lelaran Titik Tetap untuk menentukan punca bagi persamaan $f(x) = x^3 - 5x + 1$ bila $x_0 = 2$. Berikan jawapan anda tepat kepada 2 tempat perpuluhan.

[7 marks]

[*7 markah*]

QUESTION 3
SOALAN 3

CLO1   (a)   Identify order and degree for the following differential equations

Kenal pasti peringkat dan darjah bagi persamaan pembezaan berikut:

     i. $\left(\frac{d^2y}{dx^2}\right)^3 + y\left(\frac{dy}{dx}\right)^2 + y^5 = 0$

[2 marks]

[*2 markah*]

     ii. $\left(\frac{d^3y}{dx^3}\right)^2 + 3\left(\frac{dy}{dx}\right)^3 - e^x - 4 = 0$

[2 marks]

[*2 markah*]

CLO1   (b)   Use the stated method to solve the following first order differential equations:

Gunakan kaedah yang dinyatakan untuk menyelesaikan persamaan pembezaan peringkat pertama berikut:

     i. $\frac{dy}{dx}e^{-x} + e^{2x} = 0$; Direct Integration Method

[5 marks]

[*5 markah*]

     ii. $\frac{dy}{dx} + 5y = e^{-2x}$; Integrating Factor Method

[6 marks]

[*6 markah*]

CLO1   (c)   Solve the following second order differential equations:

Selesaikan persamaan pembezaan peringkat kedua berikut:

     i. $\dfrac{d^2y}{dx^2} + 8\dfrac{dy}{dx} + 15y = 0$

[4 marks]

[*4 markah*]

     ii. $\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} + 10y = 0$

[6 marks]

[*6 markah*]

QUESTION 4
SOALAN 4

CLO1   (a)   A firm manufactures two types of products, P and Q, each of which requires processing time of lathes, grinders and polishers. The processing times (hour) needed for each type of product are given in the Table 4(a) below. The total processing time available is 124 hours on lathes, 127 on grinders and 121 hours on polishers. The profit of type P is RM29 and type Q is RM19.

Express the variables, objective function and constraints that meet the description above.

Sebuah firma mengeluarkan dua jenis produk, P dan Q, masing-masing memerlukan masa pemprosesan bagi pelarik, pengisar dan penggilap. Masa (jam) yang diperlukan untuk setiap jenis produk diberikan dalam Jadual 4(a) di bawah. Jumlah masa pemprosesan yang diperuntukkan ialah 124 jam untuk pelarik, 127 jam untuk pengisar dan 121 jam untuk penggilap. Keuntungan bagi jenis P ialah RM29 dan jenis Q ialah RM19.

Ungkapkan pembolehubah, fungsi objektif dan kekangan yang memenuhi huraian di atas.

Table 4(a) / *Jadual 4(a)*

Product	Time required (hours)		
	Lathe	Grinder	Polisher
P	8	8	5
Q	5	2	2

[5 marks]

[5 markah]

CLO1 (b)   A manufacturer produces a product with two different models, x and y. Profit for Model x contributes of RM50/unit and model y, RM30/unit. Raw materials, $R1$ and $R2$ are required for production. At least 18 kg of $R1$ and 12 kg of $R2$ must be used daily. At most, 34 hours of labor are to be utilized. The objective function and constraints involving the production of Model x and Model y are stated as below:

Sebuah kilang menghasilkan satu produk dengan dua model yang berbeza, x dan y. Keuntungan bagi Model x menyumbang sebanyak RM50/unit dan model y, RM30/unit. Bahan mentah, R1 dan R2 diperlukan untuk penghasilan produk. Sekurang-kurangnya 18 kg R1 dan 12 kg R2 mesti digunakan setiap hari. Paling banyak, 34 jam tenaga kerja akan digunakan. Fungsi objektif dan kekangan yang melibatkan penghasilan Model x dan Model y dinyatakan seperti di bawah:

$$Z = 50x + 30y$$
$$2x + y \geq 18$$
$$x + y \geq 12$$
$$3x + 2y \leq 34$$

i.   Construct the graph and shade the region R that satisfies all of the stated constraints

Bina graf dan lorekkan rantau R yang memenuhi semua kekangan yang dinyatakan

[7 marks]

[7 markah]

ii.   Calculate the maximum profit for this product.

Kira keuntungan maksimum bagi produk ini.

[3 marks]

[3 markah]

CLO1 | (c)   Given a Linear Programming problem with

maximum $Z = \frac{1}{2}x + 3y$ and constraint:

Diberi satu permasalahan Pengaturcaraan Linear dengan

maksimum $Z = \frac{1}{2}x + 3y$ dan kekangan:

$$x + y \leq 40$$
$$2x + y \leq 10$$
$$x, y \geq 0$$

i.   Write the problem in Standard Simplex Form.

Tuliskan pernyataan masalah dalam Bentuk Simplex Piawai.

[2 marks]

[2 markah]

ii.   Solve the given Linear Programming problem by using Simplex Method.

Selesaikan pengaturcaraan linear yang diberikan dengan menggunakan kaedah Simplex.

[5 marks]

[5 markah]

iii.   Based on the answer from question c(ii) above, determine the optimum solution of this Linear Programming problem.

Berdasarkan kepada jawapan soalan c(ii) di atas, tentukan penyelesaian optimum bagi permasalahan Peraturcaraan Linear tersebut

[3 marks]

[3 markah]

SOALAN TAMAT

DESCRIPTIVE STATISTICS		
Number of class	Sturges Rule, $k = 1 + 3.33 \log n$	Rule of Thumb, $2^k > n$
Mean	$\bar{x} = \dfrac{\sum x}{n}$	$\bar{x} = \dfrac{\sum (fx)}{\sum f}$
Median	$\text{Median} = L_m + \left[\dfrac{\dfrac{N}{2} - F}{f_m}\right] C$	
Mode	$\text{Mode} = L_{Mo} + \left[\dfrac{d_1}{d_1 + d_2}\right] C$	
Quartile	$Q_k = L_{Q_k} + \left[\dfrac{\dfrac{kN}{4} - F}{f_{Q_k}}\right] C \qquad ; k = 1, 2, 3$	
Decile	$D_k = L_{D_k} + \left[\dfrac{\dfrac{kN}{10} - F}{f_{D_k}}\right] C \qquad ; k = 1, 2, 3 \ldots\ldots 9$	
Percentile	$P_k = L_{P_k} + \left[\dfrac{\dfrac{kN}{100} - F}{f_{P_k}}\right] C \qquad ; k = 1, 2, 3 \ldots\ldots 99$	
Mean Deviation	$E = \dfrac{\sum \left| x - \bar{x} \right|}{n}$	$E = \dfrac{\sum \left(\left| x - \bar{x} \right| f \right)}{\sum f}$
Variance	$s^2 = \dfrac{\sum \left(x - \bar{x} \right)^2}{n}$	$s^2 = \dfrac{\sum\limits_{i=1}^{n} x_i^2 - n\bar{x}^2}{n}$
	$s^2 = \dfrac{\sum \left[\left(x - \bar{x} \right)^2 f \right]}{\sum f}$	$s^2 = \dfrac{\sum fx^2}{\sum f} - \left[\dfrac{\sum fx}{\sum f} \right]^2$
Standard Deviation	$s = \sqrt{\text{variance}}$	

NUMERICAL METHOD

Crout Method	$A = \begin{pmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{pmatrix} \begin{pmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{pmatrix}$
Doolittle Method	$A = \begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix}$
Newton Raphson Method	$x_{n+1} = x_n - \dfrac{f(x)}{f'(x)}$
False Position Method	$x_0 = \dfrac{1}{y_2 - y_1} \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}$

PROBABILITY

$E = pn$	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$P(B\|A) = \dfrac{P(B \cap A)}{P(A)}$	$P(A \cap B) = P(A) \bullet P(B)$
	$P(A \cap B) = P(A) \bullet P(B\|A)$

SOLUTION FOR 1st ORDER DIFFERENTIAL EQUATION

Homogeneous Equation $y = vx$ and $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$	**Linear Factors (Integrating Factors)** $y \bullet IF = \int Q \bullet IF\, dx$ Where $IF = e^{\int P\, dx}$
	Logarithmic $a = e^{\ln a}$ $a^x = e^{x \ln a}$ $\int a^x\, dx = \dfrac{a^x}{\ln a} + c$

GENERAL SOLUTION FOR 2nd ORDER DIFFERENTIAL EQUATION

Equation of the form	$a\dfrac{d^2 y}{dx^2} + b\dfrac{dy}{dx} + cy = 0$
Quadratics Formula	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
1. Real & different roots2	$y = Ae^{m_1 x} + Be^{m_2 x}$
2. Real & equal roots	$y = e^{mx}(A + Bx)$
3. Complex roots	$y = e^{\alpha x}(A\cos \beta x + B\sin \beta x)$

DIFFERENTIATION

1.	$\dfrac{d}{dx}(k) = 0, k$ is constant	2.	$\dfrac{d}{dx}(x^n) = nx^{n-1}$ [Power Rule]
3.	$\dfrac{d}{dx}(ax^n) = anx^{n-1}$	4.	$\dfrac{d}{dx}\big(f(x) \pm g(x)\big) = f'(x) \pm g'(x)$
5.	$\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$ [Product Rule]	6.	$\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$ [Quotient Rule]
7.	$\dfrac{dy}{dx} = \dfrac{du}{dx} \times \dfrac{dy}{du}$ [Chain Rule]	8.	$\dfrac{d}{dx}(e^x) = e^x$
9.	$\dfrac{d}{dx}\left(e^{ax+b}\right) = e^{ax+b} \times \dfrac{d}{dx}(ax+b)$	10.	$\dfrac{d}{dx}(\ln x) = \dfrac{1}{x}$
11.	$\dfrac{d}{dx}[\ln(ax+b)] = \dfrac{1}{ax+b} \times \dfrac{d}{dx}(ax+b)$	12.	$\dfrac{d}{dx}(\sin x) = \cos x$
13.	$\dfrac{d}{dx}(\cos x) = -\sin x$	14.	$\dfrac{d}{dx}(\tan x) = \sec^2 x$
15.	$\dfrac{d}{dx}[\sin(ax+b)] = \cos(ax+b) \times \dfrac{d}{dx}(ax+b)$	16.	$\dfrac{d}{dx}[\cos(ax+b)] = -\sin(ax+b) \times \dfrac{d}{dx}(ax+b)$
17.	$\dfrac{d}{dx}[\tan(ax+b)] = \sec^2(ax+b) \times \dfrac{d}{dx}(ax+b)$	18.	$\dfrac{d}{dx}[\sin^n u] = n\sin^{n-1} u \times \cos u \times \dfrac{du}{dx}$
19.	$\dfrac{d}{dx}[\cos^n u] = n\cos^{n-1} u \times -\sin u \times \dfrac{du}{dx}$	20.	$\dfrac{d}{dx}[\tan^n u] = n\tan^{n-1} u \times \sec^2 u \times \dfrac{du}{dx}$

INTEGRATION

1.	$\displaystyle\int ax^n dx = \dfrac{ax^{n+1}}{n+1} + c \ \ ;\{n \neq -1\}$	2.	$\displaystyle\int (ax+b)^n dx = \dfrac{(ax+b)^{n+1}}{(a)(n+1)} + c \ ;\{n \neq -1\}$
3.	$\displaystyle\int k \ dx = kx + c, k$ is constant	4.	$\displaystyle\int_a^b f(x)dx = F(b) - F(a)$
5.	$\displaystyle\int \dfrac{1}{x} \ dx = \ln x + c$	6.	$\displaystyle\int \dfrac{1}{ax+b} \ dx = \dfrac{1}{a} \times \ln(ax+b) + c$
7.	$\displaystyle\int e^x \ dx = e^x + c$	8.	$\displaystyle\int e^{ax+b} \ dx = \dfrac{1}{a} \times e^{ax+b} + c$
9.	$\displaystyle\int \sin x \ dx = -\cos x + c$	10.	$\displaystyle\int \cos x \ dx = \sin x + c$
11.	$\displaystyle\int \sec^2 x \ dx = \tan x + c$		
12.	$\displaystyle\int \sin(ax+b) \ dx = -\dfrac{1}{\dfrac{d}{dx}(ax+b)} \times \cos(ax+b) + c$		
13.	$\displaystyle\int \cos(ax+b) \ dx = \dfrac{1}{\dfrac{d}{dx}(ax+b)} \times \sin(ax+b) + c$		
14.	$\displaystyle\int \sec^2(ax+b) \ dx = \dfrac{1}{\dfrac{d}{dx}(ax+b)} \times \tan(ax+b) + c$		